Concordia University
Department of Mathematics and Statistics

Course	**Number**	**Section**	
EMAT	233/2	Q	
Examination	**Date**	**Time**	**Pages**
Winter	December 2002	3 hours	2
Instructors			**Course Examiner**
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Special instructions

Answer all questions: **NO CALCULATORS ALLOWED**

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[10] **Problem 1.** Compute the curvature $\kappa(t)$ of the curve $\mathcal{C}$ defined by

$$\vec{r}(t) = t\mathbf{i} + \frac{t^2}{2}\mathbf{j} + \frac{t^3}{3}\mathbf{k}.$$

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[10] **Problem 2.** Consider the surface defined by

$$\cos(zx)\mathrm{e}^y = 1$$

(a) Find the equation of the tangent plane to the surface at the point $(\pi, 0, 0)$
(b) Find the equation of the normal line to the surface at the point $(\pi, 0, 0)$

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[10] **Problem 3.** Consider the function

$$G(x, y) = x^2 - \cos(yx)$$

with $x = x(u, v) = u^2 + v^2$ and $y = y(u, v) = uv$. Compute the partial derivatives $\frac{\partial G}{\partial u}$ and $\frac{\partial G}{\partial v}$. The answer must be expressed in terms of u, v **only**.

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[10] **Problem 4.** Consider the equation (D'Alambert equation):

$$\frac{\partial^2}{\partial t^2}U(t, x) - \frac{\partial^2}{\partial x^2}U(t, x) = 0.$$

(a) Prove that $U(t, x) = \mathrm{e}^{x-t}$ is a solution.
(b) Prove that if $f(s)$ is a twice-differentiable function (i.e. the second derivative exists and it is continuous) then the two functions

$$U_{(+)}(t, x) = f(x + t)\ , \qquad U_{(-)}(t, x) = f(x - t)$$

solve D'Alambert equation for both choices of the signs.

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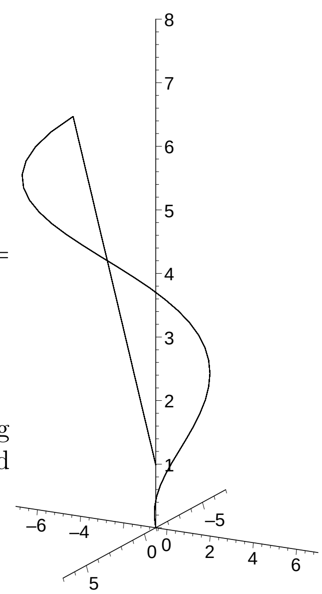
[10] **Problem 5.**



Consider the vector field

$$\vec{F}(x, y, z) = \left(2xy + yz + \frac{2x}{1 + x^2}\right)\mathbf{i} + \left(x^2 + xz\right)\mathbf{j} + \left(xy + z^2\right)\mathbf{k}\ .$$

(a) Show that $\mathrm{curl}(\vec{F}) = \vec{0}$. Can you find a function $\Phi(x, y, z)$ such that $\vec{F}(x, y, z) = \vec{\nabla}\Phi(x, y, z)$? If yes then find the most general such function.
(b) Compute the line integral

$$\int_{\mathcal{C}} \vec{F} \cdot \mathrm{d}\vec{r}$$

where the curve $\mathcal{C}$ is the piecewise smooth curve starting at $A \equiv (0, 0, 0)$ and ending at $B \equiv (0, 0, 1)$ consisting of the spiral $\vec{r}(t) = \langle t\cos(t), t\sin(t), t\rangle$ for $t \in [0, 2\pi]$ joined to the segment starting from $(2\pi, 0, 2\pi)$ and arriving at $B \equiv (0, 0, 1)$.

[10] Problem 6.

Consider the vector field

$$\vec{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + \left(x^2 z^3 \cos(x^2+y^2+z^2)\right)\mathbf{k}\ .$$

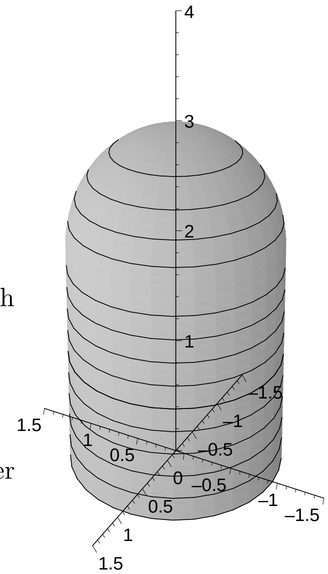
Compute the flux of $\mathrm{curl}(\vec{F})$

$$\iint_{\mathcal{S}} \mathrm{curl}(\vec{F})\cdot\mathbf{n}\ \mathrm{d}S$$

through the surface $\mathcal{S}$ consisting of a cylinder with a hemisphere on top oriented with the outward normal

$$\left\{x^2+y^2=1,\ z\in[0,2]\right\} \cup \left\{x^2+y^2+(z-2)^2=1\ ,\ \ z\geq 2\right\},$$

and with a boundary consisting of the circle at the base of the cylinder. Whatever answer you find motivate it by using the appropriate theorem.



[10] Problem 7. Using the appropriate theorem, compute the following line integral in the plane

$$\oint_{\mathcal{C}} \Big(5y + \ln\left(4+\cos(x)\right)\Big)\mathrm{d}x + \Big(2x^2y - \mathrm{e}^{\sin(y)}\Big)\mathrm{d}y$$

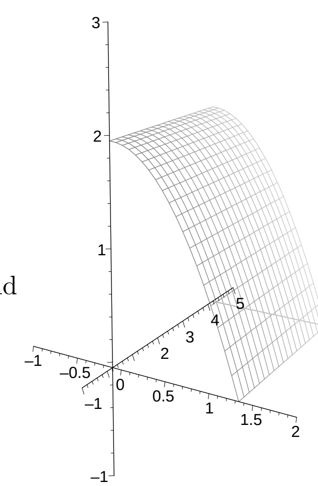
where $\mathcal{C}$ is the piecewise smooth closed simple curve oriented counterclockwise given by the boundary of the rectangle of vertices $(0,0)$, $(2,0)$, $(2,1)$ and $(0,1)$.

[10] Problem 8.

Evaluate the surface integral

$$\iint_{\mathcal{S}} xy(9-4z)\mathrm{d}S$$

where $\mathcal{S}$ is the the portion of the parabolic cylinder $z=2-x^2$ in the first octant and bounded by $x=0$, $y=0$, $y=4$, $z=0$.



[10] Problem 9. Use the Divergence Theorem to compute the flux of the vector

$$\vec{F}(x,y,z) = \left(4x + \cos(y^2+z^2)\right)\mathbf{i} + \left(\mathrm{e}^{x^2z^4} - 2y + \ln(1+z^2)\right)\mathbf{j} + \left(\cos(\ln(1+x^2+y^4))\right)\mathbf{k}$$

through the surface of the sphere of center $(1,1,1)$ and radius $R=2$ ouriented with the outward normal.
[Formulas you may find useful: (Area of the sphere of radius R)$=4\pi R^2$: (Volume of the sphere of radius R)$=\frac{4\pi}{3}R^3$.]

[10] Problem 10. Evaluate the following double integral via polar coordinates

$$\iint_{\mathcal{R}} x\,\mathrm{e}^{-\sqrt{x^2+y^2}}\mathrm{d}A$$

where $\mathcal{R}$ is the left semi-disk of radius 2:

$$\mathcal{R} = \left\{(x,y):\ x^2+y^2\leq 4\ ;\ x\geq 0\right\}\ .$$