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Corrigendum

Corrigendum to “Large deviations for multi-scale jump-diffusion processes” [Stochastic Process. Appl. 127 (2017) 1297–1320]

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In Section 3 of this paper we present some methods for checking that the comparison principle holds for the Cauchy problem for $\overline{H}_0$:

$$\begin{aligned} \partial_t u_0(t,x) &= \overline{H}_0 u_0(x, \partial_x u_0(t,x)), \qquad && (t,x) \in (0,T] \times \mathbb{R}, \\ u_0(0,x) &= h(x), && x \in \mathbb{R}, \end{aligned} \tag{1}$$

The method as stated and proved in Lemma 10 of the Appendix is incorrect. Namely, the claim that it is enough for $\overline{H}_0$ to be uniformly continuous in x, p on compact sets was insufficient. In addition to continuity of $\overline{H}_0$ one also needs to check one of the following conditions, either: (a) uniformly in x, $|\overline{H}_0|$ is coercive in p and has an x-independent modulus of continuity in p on bounded sets; or: (b) $|\overline{H}_0|$ has compact level sets. A correct version of Lemma 10 and its proof are as follows.

Lemma 1. *Let u_1 be a bounded, upper semicontinuous (u.s.c.), viscosity subsolution and u_2 a bounded, lower semicontinuous (l.s.c.), viscosity super solution of $\partial_t u(t,x) = \overline{H}_0(x, \partial_x u(t,x))$ respectively. If $\overline{H}_0(x,p)$ is continuous, and satisfies conditions in either (a) or in (b):*

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(a) for any $\tilde{R} < \infty$

$$\lim_{|p|\to\infty} \inf_x |\overline{H}_0(x,p)| = \infty, \tag{2}$$

i.e. $|\overline{H}_0(x,p)| \to \infty$ *uniformly in* x *as* $|p| \to \infty$, *and for* $|p_1|, |p_2| \leq \tilde{R}$,

$$|\overline{H}_0(x,p_1) - \overline{H}_0(x,p_2)| \leq w_{\tilde{R}}(p_1 - p_2) \tag{3}$$

uniformly in x, *where* $w_{\tilde{R}}$ *is a non-decreasing function with* $w_{\tilde{R}}(0+) = 0$.

(b) $|\overline{H}_0(\cdot,\cdot)|$ *has compact level sets, i.e.* $\forall C > 0$, $\exists$ *compact sets* $K_C, \tilde{K}_C \subset \mathbb{R}$, *such that*

$$\{(x,p) : |\overline{H}_0(x,p)| \leq C\} \subset K_C \times \tilde{K}_C. \tag{4}$$

and if $u_1(0,\cdot) \leq u_2(0,\cdot)$, *then* $u_1 \leq u_2$ *on* $[0,T] \times \mathbb{R}$ *for any* $T > 0$.

Proof. Suppose

$$\sup_{t\leq T,x} \{u_1(t,x) - u_2(t,x)\} = A > 0. \tag{5}$$

Define

$$\psi(t,x,s,y) = u_1(t,x) - u_2(s,y) - \frac{(x-y)^2}{2\epsilon} - \beta\left(x^2 + y^2\right) - \frac{(t-s)^2}{2\alpha} - \frac{At}{4T}.$$

Fix $\beta > 0$ and let $(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}, \bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha})$ denote the point of maximum of ψ in $([0,T] \times \mathbb{R} \times [0,T] \times \mathbb{R})$ for $\epsilon > 0, \alpha > 0$.

Using

$$\psi(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}, \bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}) + \psi(\bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}, \bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}) \leq 2\psi(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}, \bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}),$$

and the boundedness of u_1 and u_2 we get

$$|\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}|^2 \leq \epsilon C, \quad \text{and} \quad |\bar{t}_{\epsilon,\alpha} - \bar{s}_{\epsilon,\alpha}|^2 \leq \alpha C. \tag{6}$$

for some $C > 0$. Therefore $|\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}| \to 0$ as $\epsilon \to 0$, and $|\bar{t}_{\epsilon,\alpha} - \bar{s}_{\epsilon,\alpha}| \to 0$ as $\alpha \to 0$.

By (5), there exists $(\tilde{t}, \tilde{x})$ such that $u_1(\tilde{t},\tilde{x}) - u_2(\tilde{t},\tilde{x}) > 3A/4$. Fix β small enough such that $2\beta\tilde{x}^2 \leq A/4$. Then

$$\psi(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}, \bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}) \geq \psi(\tilde{t},\tilde{x},\tilde{t},\tilde{x}) = u_1(\tilde{t},\tilde{x}) - u_2(\tilde{t},\tilde{x}) - 2\beta\tilde{x}^2 - \frac{A\tilde{t}}{4T} > \frac{A}{4}$$

and

$$\begin{aligned}\beta\left(\bar{x}^2_{\epsilon,\alpha} + \bar{y}^2_{\epsilon,\alpha}\right) &\leq u_1(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}) - u_2(\bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}) - \psi(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}, \bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}) \\ &< \|u_1\| + \|u_2\| - \frac{A}{4}\end{aligned}$$

So for fixed $\beta > 0$, there exists an $R_\beta > 0$ such that $|\bar{x}_{\epsilon,\alpha}|, |\bar{y}_{\epsilon,\alpha}| \leq R_\beta$ for all $\epsilon > 0$, $\boldsymbol{\alpha} > \mathbf{0}$ and

$$\lim_{\beta\to 0} \beta R_\beta = 0. \tag{7}$$

We next show that one can assume without loss of generality that $\bar{t}_{\epsilon,\alpha}, \bar{s}_{\epsilon,\alpha} \neq 0$.

Claim. *Given* $\beta \in (0, \frac{A}{8\tilde{x}^2}]$, *there exists an* $\alpha > 0$ *and a sequence* $\epsilon_k \to 0$ *along which* $\bar{t}_{\epsilon,\alpha} \neq 0$ *and* $\bar{s}_{\epsilon,\alpha} \neq 0$.

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Proof. Given $\beta \in (0, \frac{A}{8\tilde{x}^2}]$, suppose for every $\alpha > 0$ there exists $\bar{\epsilon}(\alpha)$ such that for $\epsilon \leq \bar{\epsilon}(\alpha)$ either $\bar{t}_{\epsilon,\alpha} = 0$ or $\bar{s}_{\epsilon,\alpha} = 0$. Let $\bar{x}_\alpha = \lim_{\epsilon\to 0}\bar{x}_{\epsilon,\alpha} = \lim_{\epsilon\to 0}\bar{y}_{\epsilon,\alpha}$, $\bar{t}_\alpha = \lim_{\epsilon\to 0}\bar{t}_{\epsilon,\alpha}$ and $\bar{s}_\alpha = \lim_{\epsilon\to 0}\bar{s}_{\epsilon,\alpha}$ (with convergence possibly along a subsequence). Note that either $\bar{t}_\alpha = 0$ or $\bar{s}_\alpha = 0$. By upper-semicontinuity of ψ, we can choose $\alpha > 0$ small enough such that $\psi(0, \bar{x}_\alpha, 0, \bar{x}_\alpha) \geq \psi(\bar{t}_\alpha, \bar{x}_\alpha, \bar{s}_\alpha, \bar{x}_\alpha) - A/8$. Then,

$$\begin{aligned}
0 &\geq u_1(0, \bar{x}_\alpha) - u_2(0, \bar{x}_\alpha) = \psi(0, \bar{x}_\alpha, 0, \bar{x}_\alpha) \\
&\geq \psi(\bar{t}_\alpha, \bar{x}_\alpha, \bar{s}_\alpha, \bar{x}_\alpha) - \frac{A}{8} \qquad \text{by upper-semicontinuity of } \psi, \\
&\geq \psi(\tilde{t}, \tilde{x}, \tilde{t}, \tilde{x}) - \frac{A}{8} \\
&= u_1(\tilde{t}, \tilde{x}) - u_2(\tilde{t}, \tilde{x}) - 2\beta\tilde{x}^2 - \frac{A\tilde{t}}{4T} - \frac{A}{8} > 3A/4 - A/4 - A/4 - A/8 = A/8 > 0
\end{aligned}$$

which gives a contradiction.

Henceforth $\lim_{\epsilon\to 0}$ is taken along the sequence in the above claim.

Let

$$\phi_1(t, x) := u_2(\bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha}) + \frac{|x - \bar{y}_{\epsilon,\alpha}|^2}{2\epsilon} + \beta\left(x^2 + \bar{y}_{\epsilon,\alpha}^2\right) + \frac{(t - \bar{s}_{\epsilon,\alpha})^2}{2\alpha} + \frac{At}{4T}$$

and

$$\phi_2(s, y) := u_1(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha}) - \frac{|\bar{x}_{\epsilon,\alpha} - y|^2}{2\epsilon} - \beta\left(\bar{x}_{\epsilon,\alpha}^2 + y^2\right) - \frac{(\bar{t}_{\epsilon,\alpha} - s)^2}{2\alpha} - \frac{A\bar{t}_{\epsilon,\alpha}}{4T}.$$

Then $(\bar{t}_{\epsilon,\alpha}, \bar{x}_{\epsilon,\alpha})$ is a point of maximum of $u_1(t, x) - \phi_1(t, x)$ and $(\bar{s}_{\epsilon,\alpha}, \bar{y}_{\epsilon,\alpha})$ is a point of minimum of $u_2(s, y) - \phi_2(s, y)$. Since u_1 and u_2 are sub and super solutions respectively, by the definition of sub and super solutions we get

$$\frac{A}{4T} + \frac{(\bar{t}_{\epsilon,\alpha} - \bar{s}_{\epsilon,\alpha})}{\alpha} \leq \overline{H}_0\left(\bar{x}_{\epsilon,\alpha}, \frac{\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}}{\epsilon} + 2\beta\bar{x}_{\epsilon,\alpha}\right), \tag{8}$$

and

$$\frac{(\bar{t}_{\epsilon,\alpha} - \bar{s}_{\epsilon,\alpha})}{\alpha} \geq \overline{H}_0\left(\bar{y}_{\epsilon,\alpha}, \frac{\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}}{\epsilon} - 2\beta\bar{y}_{\epsilon,\alpha}\right). \tag{9}$$

For fixed α and β, inequalities (8) and (9) give us $\{\frac{|\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}|}{\epsilon}\}_\epsilon$ remains bounded for all $\epsilon > 0$, since $\frac{|\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}|}{\epsilon}$ diverging to ∞ would contradict conditions (2) and (4) of $\overline{H}_0$. Also recall that $|\bar{x}_{\epsilon,\alpha}|, |\bar{y}_{\epsilon,\alpha}| \leq R_\beta$. Keeping α fixed, take $\lim_{\epsilon\to 0}$ (possibly along a subsequence for convergence) of inequalities (8) and (9). Denote $L_\beta := \lim_{\epsilon\to 0}\frac{\bar{x}_{\epsilon,\alpha} - \bar{y}_{\epsilon,\alpha}}{\epsilon}$, $l_\beta := \lim_{\epsilon\to 0}\bar{x}_{\epsilon,\alpha} = \lim_{\epsilon\to 0}\bar{y}_{\epsilon,\alpha}$, $\bar{t} = \lim_{\epsilon\to 0}\bar{t}_{\epsilon,\alpha}$ and $\bar{s} = \lim_{\epsilon\to 0}\bar{s}_{\epsilon,\alpha}$, then

$$\frac{A}{4T} + \frac{(\bar{t} - \bar{s})}{\alpha} \leq \overline{H}_0\left(l_\beta, L_\beta + 2\beta l_\beta\right), \tag{10}$$

and

$$\frac{(\bar{t} - \bar{s})}{\alpha} \geq \overline{H}_0\left(l_\beta, L_\beta - 2\beta l_\beta\right). \tag{11}$$

Subtracting (11) from (10), we get

$$\frac{A}{4T} \leq \overline{H}_0\left(l_\beta, L_\beta + 2\beta l_\beta\right) - \overline{H}_0\left(l_\beta, L_\beta - 2\beta l_\beta\right). \tag{12}$$

Recall that by (7), we have $\lim_{\beta\to 0}\beta l_\beta = 0$.

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Under Condition 1, we have $\sup_\beta |L_\beta| \leq \tilde{R}$ by (2) and the right hand side of (12) becomes $\overline{H}_0\left(l_\beta, L_\beta + 2\beta l_\beta\right) - \overline{H}_0\left(l_\beta, L_\beta - 2\beta l_\beta\right) \leq w_{\tilde{R}}(4\beta l_\beta)$. Taking $\beta \to 0$ in (12), we get

$$A \leq 0$$

which contradicts (5).

Under Condition 2: inequalities (10) and (11) with (4) imply $\{l_\beta\}$ and $\{L_\beta\}$ remain bounded as $\beta \to 0$. Taking $\beta \to 0$ in (12), by uniform continuity of $\overline{H}_0$ over compact sets, we get

$$A \leq 0$$

which contradicts (5).

Therefore we must have

$$\sup_{t,x}\{u_1(t,x) - u_2(t,x)\} \leq 0.$$

which gives us $u_1 \leq u_2$. □

Accordingly the correct statement of Corollary 4. should also reflect this correction as follows: in part (i) $\overline{H}_0$ is continuous and satisfies either (a) or (b) of Lemma 10; in (ii) the coefficients $b_1(x,y), \sigma(x,y), k_1(x,y,z)$ are independent of x, the coefficients $b(x,y), \sigma(x,y), k(x,y,z)$ are bounded, and $\rho = 0$. The Proof of (i) is immediate from Lemma 10. The Proof of (ii) follows from that fact that the rate function J will be independent of x and p, while $|\int V(y;x,p)d\mu(y)|$ will, for any $\mu \in \mathcal{P}(R)$, be bounded by a version of V with constants in place of b, σ, and k, implying that the two conditions (a) of Lemma 10 will be satisfied.

All examples in this paper still satisfy the conditions of the corrected version of Lemma 10. Aside from the change above, we would like to correct two typographical errors: in Example 4.1 the expression for $\bar{L}(q)$ should be

$$\bar{L}(q) = \begin{cases} -a + \sqrt{a^2 + 2bq^2} - a\ln 2b + a\ln\left(\frac{-2a^2}{q^2} + \frac{2a}{q^2}\sqrt{a^2+2bq^2}\right) & \text{if } q \neq 0 \\ 0 & \text{if } q = 0 \end{cases}$$

and in Example 4.2 the expression for $\overline{H}_0^{PDMP}$ should have an additional term $(e^p + e^{-p})/2$.