Concordia University

EMAT 213 - Final Exam

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Date: December 2005.

Time allowed: 3 hours.

Directions: non graphing calculators are allowed.

Note: Marked out of 100 points

[30 pts] Problem 1

Solve the following *first* order ODEs

(a) $$\left(2x\cos(x^2+y^2) - 4x^3y\right)\mathrm{d}x + \left(2y\cos(x^2+y^2) - x^4 + 2y\mathrm{e}^{y^2}\right)\mathrm{d}y = 0$$

(b) $$y' = \frac{\mathrm{e}^{y+5x+3}}{y+5x+3} - 5$$ [*Hint: with a substitution* $u = Ax + By + C$ *it becomes separable*]

(c) $$\sqrt{x^3+1}\; y' = \frac{x^2}{y^2}\ .$$

[20 pts] Problem 2

Solve the following linear ODEs by finding the general solution.

(a) $$y^{(7)} - 6y^{(6)} + 20y^{(5)} - 56y^{(4)} + 112y^{(3)} - 160y'' + 192y' - 128y = x\ ;$$

Note : $m^7 - 6m^6 + 20m^5 - 56m^4 + 112m^3 - 160m^2 + 192m - 128 = (m-2)^3(m^2+4)^2$

(b) $$x^2y'' - 2y = x^2\ , \qquad (x > 0)$$

Note : variation of parameters [put it in standard form!].

[10 pts] Problem 3

Solve the following nonlinear ODE of second order

$$(y+2)y'' = (y')^2$$

using the substitution $y' = u(y)$, $y'' = u\frac{\mathrm{d}u}{\mathrm{d}y}$ (variable x is missing).

[10 pts] Problem 4

Find the general solution as a power series centered at the regular point $x = 0$ of the differential equation

$$y'' + x^2y = 0$$

Give only the first six terms of the series (namely up to degree 5).

[10 pts] Problem 5

Solve the following *nonhomogeneous linear* system of ODEs

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x + 2y + t \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = x + 1 \end{cases}$$

by finding X_c (complementary solution), X_p (particular solution).

[10 pts] Problem 6

A cake is removed from an oven at a temperature of 150 degrees (Celsius) and left to cool down on a countertop in a room at a temperature of 20 degrees. After 2 minutes the temperature of the cake is 120 degrees. After how many minutes (approximately) the cake's temperature will have dropped to 40 degrees?

[15 pts] Problem 7

[10 pts] A spring of constant $k = 2N/m$ is immersed in a viscous fluid. A mass of one Kilogram is attached to the loose end and it is released half a meter below the equilibrium position without any initial velocity. Find the motion of the mass knowing that there is a damping coefficient equal to $2\ N \times s/m$.

[5pts] The viscous fluid is removed. The mass is now subject to an external force

$$F_{ext} = 0.001 \sin(\gamma t) \text{ Newtons.}$$

How should we choose γ such as to break the spring in the long run? **Explain**.